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On the Brillinger-Mixing Property of Stationary Point Processes with Some Applications

Abstract

We consider a stationary point process (PP) $N = \sum_{i\geq 1} \delta_{X_i}$ on $\mathbb{R}^d$ satisfying in addition that, for each $k \geq 2$, the reduced kth-order factorial cumulant measure $\gamma^{(k)}_{red}(\cdot)$ has finite total variation $\|\gamma^{(k)}_{red}\|$ on $\mathbb{R}^{d(k-1)}$. Such property of a random counting measure $N(\cdot)$, which is attributed to D. R. Brillinger, expresses weak mutual correlatedness between the numbers of atoms of N in distant sets. This condition is essential to prove asymptotic normality of shot noise processes, moment estimators, empirical product densities etc. The aim of the talk is to compare Brillinger-mixing with other notions of mixing (from ergodic theory). Correcting a result by G. Ivanoff (1982), we first prove that Brillinger-mixing implies the usual *mixing* (even of any order) of N provided that the distribution of the random number $N([0,1)^d)$ is uniquely determined by its moments. Furthermore, we present a family of Brillinger-mixing PP's which is neither mixing nor ergodic. If, in addition, $\|\gamma^{(k)}_{red}\| \leq a^k\, k!$ for some $a > 0$ and any $k \geq 2$ (N is said to be *strongly Brillinger-mixing*), we show that the tail-σ-algebra of N is trivial. Finally, we give simple conditions ensuring (strong) Brillinger-mixing of stationary α-determinantal PP's and apply this properties to show among others asymptotic normality for kernel estimators of the second product density of this class of PP's.